Wittgenstein's Concept of Identity

Wittgenstein represented in the early 30s in Cambridge a new perspective on mathematical logic, which should lead to better reliability and applicability of mathematics. Not only the concept of 'grammar' arises for the identity-sign, which triggers the horror of the Trinity Mathematical Society in 1930 and of E. G. Moore 1932. But additionally Wittgenstein does not use formal language to treat problems of mathematical logic. Otherwise, his formal conceivement of the concepts becomes obvious, if the early criticism in the Notebooks of 1914 and the Tractatus Logico-Philosophicus is taken into account. His idea how to modify logical leads to an operational use of identity constants as an assignment operator. This becomes obvious in the thirties. But at that time, he intermingled methodologically formal concepts and rather hermeneutical concepts. Hence, that he speaks about incompleteness in Gödel's sense, and not merely of 'unvollständig' in a very general sense, has to be noticed by awareness for his methodology, to ensemble concepts from different contextual realms together, to create tensions that initialize a comparative understanding and an improved concept formation. Within the 30s, this improvement focused on the use of identity and quantification, which Wittgenstein changed into a sign for coding like an assignment operator in informatics. Special examples he gives for the reduction of logical constants, and the operational and visual criteria he developes for the correct application of a rule show how he provided the foundations for the genuine Turing-machine within mathematics.

1 "Still the Wittgenstein of the Tractatus"

Wittgenstein was indulged with acceptance and compliments by the Cambridge mathematicians of the 30s, not although, but because he had already started to think of philosophy in a new manner. For different reasons, in a way represented by Russell, Gödel and Schlick, Wittgenstein's philosophy was in his special approach to logic, mathematics and language the most promising of all. After the acceptance of his TLP as his thesis for a Ph.D. degree, and an obligatory oral examination with Russell and Moore on the 18th June 1929, he became lecturer and fellow of Cambridge's Trinity College.

1.2 Identity and quantification in the TLP

To understand how he confirmed the expectations, we should have a look how he discussed problems of identity and quantification since 1914.

In Ryle's library, which Ryle inherited to Linacre College in Oxford, a five-pages typescript can be found, in which comments on every single paragraph of Wittgenstein's "Notes on Logic" are given, to compare them with the TLP. The comments are sometimes merely allusive and expressive ("Carnap!", "interesting!", "Already in the T.", "Better than Vienna!"), hence, the typescript was just for personal use and exchange with friends. Two hand-written remarks "shows", "88 argument against?" are not by the same hand. Another typescript in Ryle's papers at Linacre with personal remarks by his hand shows, that both were written on different type-writers or at least by different secretaries. However, this might be due to the distance in time. The content of the Comments on Notes shows excellent acquaintance not only with Wittgenstein's philosophy but with his philosophical context - on the other hand, comments like "Already in the T." are astonishing. To perceive a Carnap-like thought in Wittgenstein's Notes on Logic (1913) points to Ayer, who was especially interested in the Vienna Circle. (Gilbert Ryle Notes and Letters. 3 Tractatus material. Typescript titled "Comments on Notes", five pages, undated, unsigned. In Linacre College, Ryle Cabinet, Oxford.)

The comments point out Wittgenstein's approach to symbols. Passing the theory of atomic sentences and their individual signs, the commentator emphasises the difference to Russell and Frege, that Wittgenstein argues against "function" and "individual" as primitive notions, that he discusses without clear result the correctness of "-" (TS "Comments on Notes", Ryle Notes and Letters, 3 Tm, Linacre, first page), and tries to give up transitivity as a result of Russell's and Frege's relation-pleasureness, and finally, comments on paragraph 17:

> "This is arbitrary, but important. He thinks "(x) x=x" is a statement of physics! see correspondence). We can draw inferences from a universally quantified expression, too. This seems to be an ad hoc (and so arbitrary) way of avoiding criticism."(relates to TLP 4.241 ff)
>
> "Yes. Inter[e]sting. General sentences are different from ab-functions. He regards general statements such as "(x)x=x" as genuine" (TS "Comments on Notes", Ryle Notes and Letters, 3 Tm, Linacre, fifth page).

The comment furthermore explains on paragraph 19, that the concept of a "Spielraum" of truth-values as the meaning of a sentence isn't developed entirely in the Notes, but at least in its beginnings (TS "Comments on Notes", Ryle Notes and Letters, 3 Tm, Linacre, first page) .

In the TLP, Wittgenstein developes the idea of a room of possibilities of truth values - to be possibly false and to be possibly true for atomic sentences, and likewise for their logical combinations in dependence of their atomic sentences (with non-trivial truth-values). Like the equivalence of sentences, that include identical sentences (not just the same truth-values, but the same expression), we think usually of the identity of a sign as a necessary truth. Like the equivalence of identical sentences $A \leftrightarrow A$, a=a should be a necessary truth, or, in Wittgenstein's TLP, a tautology which is always true, as the truth values which are assigned to the connected sentences by the logical connective "$\leftrightarrow$" are always the same. The connective "$\leftrightarrow$" is conceived to be reduced to $\top$ or just four truth value positions.
But necessity, as a property of a sentence, $\Box$ F, would contradict an unrestricted theorem of modal

consistence like (◊ F and ◊ ¬ F), as (□ F and ◊ ¬ F) exclude each other by definition.[1]
Hence, Wittgenstein has to reject necessity generally, if he wants to keep a modal realm for every sentence. His solution for logically *connected sentences* is to give up intensionality for tautologies. Synthetical truth, which relates the syntactical transformations within equivalence to the realm of experience and possibly existing worlds, has to be given up for formal truth, with completely depends on the rules for the connectives and the genuine empirial truth or falseness of the connected atomic sentences.

In a letter to Moore, probably written on 23 Oct 1913, Wittgenstein gives Moore his address in Skjolden and complains: "Identity plays hell with me!" (Moore, Papers and Correspondence, Cambridge University Library, Department of Manuscripts and University Archives, MS Add.8330 8 W/32/2)

One aspect, why identity played hell with him, is the problem to transfer the reductive concept of tautologies to an expression like a=a, which is hopelessly synthetic, as "a" should be a denoting variable. Furthermore, as not sentences, but signs-as-objects are connected by the logical *constant* "=", there are no truth-values to which the truth of this 'tautology' might be reduced. And furthermore, a=a seems to imply, that there is an a that is identical. Hence, regularly in formal languages, the generalization of identity is allowed (Link, Godehard. Collegium Logicum, Bd.1 mentis, Paderborn 2009: 359)[2]

a=a -> Ex x (x=a).

Hence, when we conceive of a=a as synthetic and necessarily true ("trivial"), we have

□ (a=a)

and a sentence follows with generalization:

Ex x (x=a)

But this sentence is contingent in respect to modality:

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1 We find as interdependent definition of possibility in Link, Godehard. Collegium Logicum, Mentis Paderborn 2009, 61: " ◊ F :↔ ¬ □ ¬ F ", what leads to □ ¬ F -> ¬ ◊ F (with negation on both sides and reduction) and hence to ¬ ◊ F and ◊ F, what is not equivalent to ◊ F and ◊ ¬ F but obviously a contradiction. Link uses the modal contingence ◊ F and ◊ ¬ F as reduction to a merely possible truth of F: (◊ F and ◊ ¬ F) ↔: ◊ F. Hence, necessity is excluded if possibility is excepted (Link 2009: 61).

2 Theorems of identity which are allowed in predicate logic with identity are: I.1 t=t (selfidentity); I.2 s=t ↔ t=a (symmetry); I.3 s=t ∧ t=r → s=r (transitivity); I.4 s=t (φ[s] ↔ φ[t]) (Leibniz), I.5 Ex x (x=x) (models not empty), I.6 Ex x (x=t) (x ∉ FV(t)) (names denote)" ["FV"= x in not a free variable for t], Link 2009: 359. Especially I.6 is our 'rule of quantification'.

◊ Ex x (x=a) and ◊ ┐ Ex x (x=a)

Now the following problem arises:

Maintenance of truth: We should think of a sentence, that is infered from a necessary truth with the help of a theorem (like generalization), that it is necessarily true , too - otherwise we would loose the special realm of truth-values we have initially assumed, hence, the maintenance rule of logic would be violated. An example provides e.g. the use of Reflection and Necessitation for the quite similar problem of disquotation and Tarskian equivalences in Halbach, Volker "Modalized Disquotationalism", in Principles of Truth, Eds. Halbach, V. and L. Horsten, Hänsel Hohenhausen, Frankfurt a.M. 2002: 75-101.. For Halbach, (even) "all uniform Tarskian equivalences are necessary" (Halbach 2002: 81).

The consequence is obvious, with disquotationalism we have immediately a complete Lord of the Rings environment:

1. □ (Bilbo = Bilbo) (Self-Identity I.1 & T-maintenance)
2. □ Ex x: x = Bilbo (Models exist I.5, Names denote I.6 & T-maintenance)
3 □ ("There is something that is Bilbo and exists" is true ↔ There is something that is Bilbo and exists) (Disquotation & T-maintenance)
4 □ Bilbo exists (with 3 & & T-maintenance)

Within a modal world and the contingence of empirical states, Bilbo vanishes again after causing a contradiction by closing his eyes for the last time:

5 ◊ ¬ Bilbo exists and □ Bilbo exists.

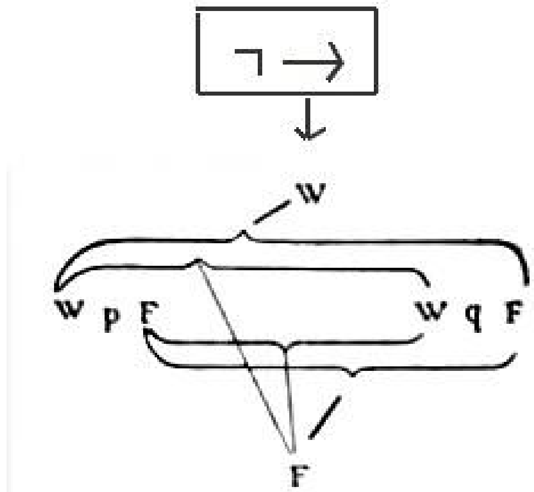
Wittgenstein saw this problem already in 1914 and never developed a complete formal theory of modality. Already in the TLP the modality is just discussed as way to conceive statements - even theorems - and when tautologies are concerned he tries to give it up for a reductive nominalism. But both concepts are not clear, and more probably given up for a third version, that emerges within his attempt to reduce logical connectives and to handle identity.

In 1914, in a notebook - entry dated 11.11.1914, he can not find a satisfying solution for the handling of identity:

> "Da a=b kein Satz, x=y keine Funktion ist, so ist eine Classe ẋ(x=x) ein Unding und ebenso die sogenannte Nullklasse Man hatte übrigens immer schon das Gefühl daß überall da wo man sich in Satzconstructionen mit x=x, a=a, etc. half, daß es sich in allen solchen Fällen um ein sich-

herausschwindeln handelte, so wenn man sagte, "a existiert" heißt "(Ex x) x=a,
Dies ist falsch: da die Definition der Klassen selbst die Existenz der wirklichen Funktionen verbirgt."

("As a=b is not a sentence, x=y no function, so a class $\dot{x}$(x=x) is a non-object and likewise the so-called empty set. Beside, we have always had the feeling that to help oneself with constructions of sentences like x=x, a=a, a.s.o. everywhere it was because one wanted to dissemble something, as when we say, "a exists" is named "(Ex x) x=a.
This is wrong: as the definition of the classes itself hides the existence of the functions, which are really there").



p	q	p -> q	not (p -> q)
w	w	w	f
w	f	f	w
f	w	w	f
f	f	w	f

(Drawing by Wittgenstein, TLP 6.1023; **truth values and logical constants by UR**)

The way he wanted to substitute the logical symbols, the logical connectives for sentences and the identity sign for names, was again inspired by physics. The idea of something that a symbolism and physics might really have in common remained in a way philosophically undefined. The meaning of a logical connective like modus pones was just conceived as a rather graphical distribution of truth-/false- signs, that reminds on wire circuits. Hence, the connectives, e.g. his example in 6.1023, the negation of the modus ponens should not any longer be signified by an arrow "→" but by the distribution of respective truth-values to the elementary propositions and to the respective logically concatenated sentences, which he wanted to mark with brackets (usually known as the "Ab-functions", comp. TS "Comments on Notes", Ryle Notes and Letters, 3 Tm, Linacre, introduction, first page).

In the design that accompanies 6.1023 TLP, Wittgenstein declares to show how he proves tautologies, but efficiently shows a system how to reduce the logical constants to truth-values, and, finally, how to create a chain of truth-maintenance, that is based on the logical combination of the negation of a modus ponens with an assignment of the resulting truth to the first sentence in the logical connexion. ($\neg$ (p $\rightarrow$q)). As $\neg$ (p$\rightarrow$q) is just true if p is true and not q[3], the result W($\neg$(p$\rightarrow$q)) ["W" for german "wahr", english "true"] of the ($\neg$ (p$\rightarrow$ q)) can be used as a continuing distribution of truth-values, in especially, as an assignment of truth to p_n and just p_n in the next 'circuit', which keeps "no wire" or "false" as the regular state:

w($\neg$($p_0 \rightarrow q_0$)) attached to p_1 is sufficient as specification to get a new functioning circuit
w($\neg$($p_1 \rightarrow$ q_1)) and so on

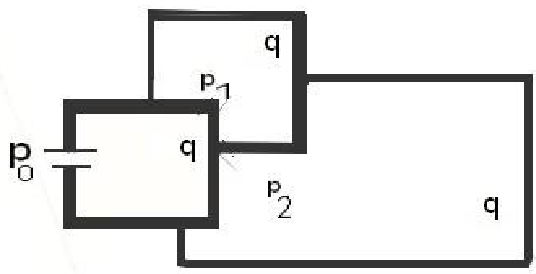
In the same way, we could attach (assign) the truth of ($\neg$($p_0 \rightarrow q_0$) to p_1, to get w($\neg$(p_1 $\rightarrow$ q_1).

This special (very interesting) example Wittgenstein presents in the TLP to show his idea of reducing the logical constants (like "$\neg$" (not) and "$\rightarrow$") to such truth-value distributions, leads to the idea to reduce the attachment of truth-values to sentence-combinations that are true. This was a most interesting way to an application, not just towards an analysis of a passive nature, but to a creative handling of this vivid logic in nature, that finally Turing succeeded to detect and to build.

Quite obviously, the importance of Turing's idea of computational states is the formal modeling of the physical, electronical structure of relais with closed and open circuits, where a p is true if power is substituted by an additional source or a primary, closed circuit. Wittgenstein's example of a negation of a modus ponens is in this sense significant, as it allows a dual structure of a given negative state and punctual modification of truth assignations, that are transponable to make theoretically infinite succeeding and depending propositions of this kind true (circuits to be closed). Even if it is easy to accept a simple model, where p is the source of wire and q is just the state "no additional wire", and a circuit becomes a source of wire if it is closed ($\neg$ (p$\rightarrow$q) and p true), it is obvious what is the unnegotiable difference between physics and logic: We need more and more wire to maintain our truth-state, whereas logic does not need any power at all and can even be true if every atomic sentence is false. This shows, that the truth of a tautology neither presupposes a propositional content nor the truth of the atomic sentences it correlates. Nevertheless the tautology has a truth-value, that is attached or assigned to it via the rule of truth-value distribution (the truth-function) that substitutes the respective logical connective. The truth-functions can already be seen

3 $\rightarrow$[p(w,**w**,f,f); q(w,**f**,w,f)] = (w,**f**,w,w) and not(w,f,w,w) = (f,<u>**w**</u>,f,f). Hence, p(w) and q(f) maintains truth in a sequence of not[$\rightarrow$, p,q].

as rules of assignment.



(Design by UR)

This is the similarity of ideas of Turing and Wittgenstein. And whereas the interest in the structure of nature was generally quite popular, promoted for example by Eddington, the idea to look at a true/false schema as application-like is reducable to Wittgenstein and Turing, as far as we know. Furthermore, what Wittgenstein alludes to - perhaps he remembered a physical model he saw or created as student of engineering in Manchester - is a conditional branching, a process to proceed automatically from one state to the other, based on binary functions.

3 Wittgenstein and Turing - similar ideas

As a philosopher, Wittgenstein didn't stop to care about the problem of the truth-functional ambiguity of some seemingly trivial propositions between necessity and possibility or beyond this realm, and to approach his 'third' solution of assignment rules without making it really explicit. Thus in his lectures on the philosophy of mathematics in 1932 (the so-called lectures "Philosophy for Mathematicians"), he declares the sentences of the excluded middle and contradiction to be "arbitrary" (Wittgenstein's Lectures. Cambridge, 1932-1935. Ed. A. Ambrose, Basil Blackwell: Oxford 1979, p. 71).

We see him, on the other hand, in his diary, occassionally concerned with mathematical problems that belong to the infinitesimal calculus, like in his pocket book MS 156, jpg 16:

> "Für jedes d gibt es e, sodass ..." z.B. if in jedem d gibt es ein e, so daß e < d. Der Beweis hiervon ist die Lösung der Ungleichung, etwa e = 1/2 d. (probably 1932-33 / 1933-34 (Wittgensteins Nachlass, Bergen electronic edition, 2000, MS 156, undated)

This formula is regularly used within the infinitesimal calculus and was of concern for Turing in his discussion of John v. Neumann. Turing uses the epsilon-equation, too, to describe a difference's minimum of periodical numbers (Turing, A. "Equivalence of left and right almost periodicity". The

Journal of The London Mathematical Society, Vol 10/4 October 1935, p. 284-285).

Wittgenstein's remark is - however it was related to Turing - not an accident. He was requested by the Cambridge mathematicians to think about their subjects and they regarded this to have been done already in parts within the TLP. How they roped him into the discussion is evident by the meetings and memberships he joined, how he presented himself to his students and with whom he liked to discuss about what.

He was elected to member of the Trinity Mathematical Society (Ludwig Wittgenstein : public and private occasions. Wittgenstein, Ludwig, 18;89-1951. Ed. Klagge, J. C. & Nordmann, A. Lofanham Oxford : Rowman & Littlefield Publishers, 2002 : 373). Furthermore, he regularly discussed the foundations of mathematics with a good friend of Russell, M.H.A. (Max) Newman (1897-1984) of St. John's College, Cambridge. He was a lecturer in mathematics and promoted Russell's Principia Mathematica (PM) with great emphasis. In a memoir, the Bloomsbury writer Frances Partridge recalled that '*the distinguished mathematician, Max Newman, who had several discussions with [Wittgenstein], told me that once - when philosophy versus mathematics was an issue between them - he heard Wittgenstein muttering under his breath, 'You ought to have been drowned at birth!*' (Frances Partridge, Memories 1981, p. 159 cit. (Klagge & Nordmann 2002: 373))

His correspondence with his student Alice Ambrose shows how much Wittgenstein liked to be accepted as philosopher of mathematics. In his letter from Austria to her, answering her letter in which she invites him to dinner and sends him notes of four lectures, he emphasises his lectures on mathematical subjects. ((McGuinness, Brian (Ed.). Wittgenstein in Cambridge. Letters and documents1911-1951. Wiley-Blackwell, Oxford 2012: 232).

Thus, the mathematicians not only accepted him, but he felt competent to discuss their most vibrant subjects. Wittgenstein's letters to Walton, a friend who left England to America, show that Wittgenstein was even sympathetic with Hardy (McGuinness 2012: 194):

> Nr. 142 To W.H.Watson 30.10.1931
> "Trin. Coll. 30.10.1931
>
> Prof. Hardy the Mathematician who was lecturer here when I was an undergraduate and then went to Oxford has now come back here as a Prof. and is one of the few people I see. I like him very much. The otherday I met Priestley (another excellent man) and we talked of you." (McGuinness, 2012: 194).

Furthermore we know, that Littlewood, the other influencial mathematician in Cambridge, judged Wittgenstein to be of the utmost importance - though his comments on Wittgenstein might have not satisfied Wittgenstein himself, who was for sure convinced (correctly) to work on something very individual, new and important, and Littlewood just evalues him as destined to become "*eventually part of logical thought*". It is quite sure that Wittgenstein thought this to be sure and not just something partially. Littlewood reports in a letter to the Council written on the 01.6.30, that he had held "6 or 8 sessions of 1 to one hour and a half" with Wittgenstein, and evalued exactly "his work is of the highest importance". Some of his ideas might seem to him "*clearly destined to become eventually part of logical thought*". But he was himself "*only an amateur in logic, even in mathematical logic*". (McGuinness 2012: 187)

The recommendation gives evidence, that Wittgenstein was coherently judged to be a modern logician. This was not a misunderstanding. To his friend Watson he recommended still Frege, even while he already thought about a different way his philosophy of mathematics should take, between 1931 and 1932, and saw an obvious difference between the more technological mathematics within the United States and the philosophical approach to mathematics and its philosophy in Cambridge. Hence, he discusses the opportunity to visit Watson in the US and rejects it, because a University abroad would not be interested in him:

> "Nr. 143
> from W.H. Watson to W.
> 'following your advise I have been dipping into Frege's 'Grundgesetze' '"(McGuinness 2012: 195)

> "Nr. 147
> Wittgenstein to Watson, 8.iv.32
> US[...] I don't know why they should ask me. All the more as Philosophy in most places there - if I'm not mistaken - takes a radically different turn from that which I care to give it." (McGuinness 2012: 199)

Likewise, he in a way confirmed the perspective english philosophy and the Vienna Circle had on him by introducing himself to Cambridge with a talk at the Aristotelian Society in Nottingham on Wednesday, July 1929, discussing "Some remarks on logical form" and inviting Russell to support him (Letters to Russell, Keynes and Moore, Ed. G.H. von Wright, Blackwell Oxford 1974: 99), but in another way explicitly notes in his later published papers Philosophical Remarks, in the same year, that "phenomenological language, 'primary language', as I have called it, does not appeal to me as a goal; now I no longer consider it necessary" (Lock, Grahame. "Some Comments on Analytic Philosophy and Phenomenology", in Logos of phenomenology and phenomenology of the logos, World Congress of Phenomenology. Ed. Anna-Teresa Tymieniecka. Oxford 2004: 51):

> "Dear Russell,
> On Saturday the 13th I will read a paper to the Aristotelian Society in Nottingham and I

would like to ask you if you could possibly manage to come there, as your presence would improve the discussion immensely and perhaps would be the only thing making it worth while at all. My paper (the one written for the meeting) is "Some remarks on logical form", but I intend to read something else to them about generality and infinity in mathematics which, I believe, will be greater fun*. - I fear that whatever one says to them will either fall flat or arouse irrelevant troubles in their mind and questions and therefore I would be much obliged to you if you came, in order - as I said - to make the discussion worth while. Yours ever L.Wittgenstein

*though it may be all Chinese to them" (Ludwig Wittgenstein, Cambridge letters : correspondence with Russell, Keynes, Moore, Ramsey and Sraffa. McGuinness, Brian ; Wright, Georg Henrik von, Blackwell: Oxford 1995: 99)

Von Wright comments "The Joint Session of the Aristotelian Society and the Mind Association was held in University College, Nottingham 12-15 July 1929. Wittgenstein's written contribution 'Some Remarks on Logical Form' was published under the title in the Supplementary Volume IX to the Proceedings of the Aristotelian Society for 1929, pp. 162-171. Russell's Appointments Diary does not show that he went to Nottingham. (McGuinness & von Wright 1995: 99)

Wittgenstein's involvement into mathematical logic and what he thought this to be is further documented by the fact, that he gave lectures not only within philosophy or for students of philosophy, but in the terms 1931-1932 explicitly for "mathematicians".

"Michaelmas 1932 (M32), (M32m), Lent 1933, Easter 1933
two sets of lectures one set "Philosophy", and another set called "Philosophy for Mathematicians" marked as (...m) (Klagge & Nordmann 2002: 344)

In Klagge & Nordmann a letter is quoted, from Wittgenstein to Mary Cartwright[4], a student of Hardy and collaborator with Littlewood's (McGuinness 2012: 207), in which Wittgenstein asks her if she allowed the use of her manuscript within his class. It shouldn't be neglected that Wittgenstein was especially polite to Lady Cartwright, who was in a way the leading figure of a club of mathematicians, the Pythagoreans, and participated in his lectures as a student of Hardy. The titel of her talk was 'What is Three?' and she reported Wittgenstein to having kept saying "give me time". He disappointed her a little by discussing "just infinite cardinals" (McGuinness 2012: 207), a conjecture that might show the emerging Americanism of Cambridge. Prof. George Temple confirmed her impressions to be a little bit disappointed, but L.C.Young took as different view, that "insight in Wittgenstein's thinking made them better mathematicians". In this sense it was accepted when Wittgenstein talked about 10 lines, Hardy's introduction of irrationals, without pointing to numbers explicitly, and in this way he was judged to "having cured Ramsey of thinking purely mathematically about logic". The math-students in Cambridge spoke generally of an "atmosphere of

4 Mary Cartwright was the later mistress of Girton and FRS, lived from 1900 to 1998.

new creativity" and within number theory a "general attitude that was parallel to modern topology" (McGuinness 2012: 207).

What was of obvious need in the thirties, was a method or an instrument, that simplified practical calculations. Though simple calculators existed, for complicated calculations charts were used. Hodges sees a gap of hundred years between Babbage's first idea of a universal machine, following a conditional branching, and Turing's Computable Numbers, which he regards as the first solution for this problem (Hodges, Andrew. Alan Turing: The Enigma. Vintage, 2012: 298). Turing's line of development was technology, Wittgenstein's simplification for a better surview of structure. Thus, if we look at Alan Turing's article about the left-right-periodicity of von Neumann, the distance between the sophisticated formalism and the difficulties with practical calculations is obvious. The way Turing shows the equivalence of left-and right-periodicity uses a difficult sequence of substitutions, and obviously prefers a high level of generalisation (e.g. into the realm of complex numbers, Turing, Alan. "Equivalence of left and right almost periodicity". The Journal of The London Mathematical Society, Vol 10/4 October 1935: 284-285) instead of using something more calculable like a dissolution of the magnitude into signed terms a.s.o. , to show the symmetry in a more simple way.

Wittgenstein was at least in some aspects 'in between', that means, he didn't think explicitly about matching into the expectations of students like Cartwright had on him, but at least within the early thirties, he didn't deny it, too.

A letter to Russell testifies this, dated Cambridge, April 1930, in which he is "Still in the motorcar to Penzance" and he explains his notation in a MS to Russell:

> "I use the sign II'. Now first of all I must say that where you find two capital I like this II this means II [II]for I had no II on my typewriter. Now II' is a prescription derived from the prescription II (i.e. the prescription according to which we develop the decimal extension if II) by some such rule as the following: "Whenever you meet a 7 in the decimal extension of II, replace it by a 3" or "Whenever you get to three 5's in that extension replace them by 2" etc. In my original M.S. I denoted this sort of thing by II 5->3 [...]"(McGuinness & von Wright, 1995: 100-101)

The four dates of lecturing and discussions, a lot of correspondence and some regular meetings with other scientists like Newman (Klagge & Nordmann 2002: McGuinness 2012: 230-241; Alice Ambrose and Morris Lazerowitz (eds) Ludwig Wittgenstein: Philosophy and Language. London: Allen & Unwin, 1972: 22-4) and Sraffa (an economist supported by Keynes and refugee from Italy's facism, fellow of King's college whom Wittgenstein met regularly since 1930) were in 1929 up to 1931 supplemented by meetings of the Cambridge Moral Science Club. The regular meetings of the undergraduates and fellows, with E.G.Moore in the chair, were opportunities to talk and discuss the latest topics, to whom the Club invited mostly fellows of Cambridge and Oxford, thus graduates and lecturers, but students like Turing, too. The programm avoided to draw a clear line between the Moral Sciences, Natural Sciences and Mathematics.

Wittgenstein started to attend the meetings on Mai 10th 1929, and stopped going into CMSC meetings beginning with the fall of 1931. Klagge suggests, that "'apparently this was because some people objected that he dominated the discussions (Wittgenstein 1995, p. 271, Wittgenstein letter to Russell, apparently from November 1935)'. Fania Pascal attended CMSC during that time, and recalls:

> 'Wittgenstein was the disturbing (perhaps disrupting) centre of these evenings. He would talk for long periods without interruption, using similes and allegories, stalking about the room and gesticulating. He cast a spell [...] Once he said: 'You cannot love God, for you do not know him', and went on elaborating the theme." (Fania Pascal 1973, in: Klagge & Nordmann, 2002: 334)

Already in 1930 Oct 23 (Letter by Richard Bevan Braithwaite to George Edward Moore; Moore, G. Edward, Personal Papers and Correspondence, Cambridge University Library, Department of Manuscripts and University Archives, MS Add.8330 (8 B/23/2)) some students feel that Wittgenstein is too prominent in GEM's discussion class.
A seven pages Moore-Document might point to an argument Wittgenstein and Moore had about Wittgenstein's use of grammar (probably the identity sign) in autumn /fall 1931, though the respective manuscript by Moore is dated 26 Februar 1932. (Lecture or statement without a title, seven pages, dated 26 Feb 1932, Cambridge University Library, Department of Manuscripts and University Archives. MS Add.8875 15/10). The further correspondence Moore-Wittgenstein shows only a discussion 1933[5], and the remark about the identity sign in the letter from 1913 already quoted.

Anywhere, he retained the possibility to visit the Club in case of a special referee. This is discussed by him in a letter to Russell written in November 1933 presumably (McGuinness 2012: 103-104).

Thus, we can take into account that it was obviously possible for Wittgenstein to attend the meetings. This is of importance for the first talk Ayer gave at the CMSC. This is reported by Ayer to have been in fall or winter 1932, and he proudly says about his talk that Wittgenstein was not only within the audience but supported his view decisively:

5 Ludwig Wittgenstein, letter to Moore, dated 1933 Oct. will not come to tea on Tuesdays, owing to GEM's lack of friendliness on two occasions . Cambridge University Library, Department of Manuscripts and University Archives. MS Add.8330 8 W/32/25; Ludwig Wittgenstein, letter to Moore, [1913 Oct 23?], Skjolden, giving address, "Identity plays hell with me! Please ask Russell whether he has got my letter". Cambridge University Library, Department of Manuscripts and University Archives .MS Add.8330 8 W/32/2*

> "Soon after the examination for greats was over, Gilbert Ryle drove me to Cambridge in order to introduce me to Wittgenstein. [...] So fas as I was concerned, the Wittgenstein whom I was meeting in the summer of 1932 was still the Wittgenstein of the Tractatus. [...] On the two or three occassions, in the course of the nineteen-thirties, when I visited Cambridge to read a paper to the Moral Science Club, he attended the meeting and took my part, usually finding in my paper more points of interest than I had myself been aware that it contained. I had the impression also that he liked me personally. He is, indeed, reported to have said, 'The trouble with Freddie Ayer is that he is clever all the time,' but I think that if he ever did say anything of this sort, it must have been much later when, as I shall relate, he too readily believed that I had been disloyal to him." (Ayer, Alfred J.. Part of My Life. London: Collins 1977: 120-121)

The date of 1932 is supported by a letter, written in February 1933, which shows Ayer to stay in Vienna. He informs Ryle about his meeting with Schlick and that it was said of Wittgenstein that he had started something new, that he is hold as "a new Pythagoras", that "Weissmann is the high priest of the cult", and that Quine had visited the Vienna Circle, with a book about his "new symbolism" to appear in summer (Wien IV, Schönburgstraße 25, bei Ines, February 19th, 1933).[6]

The date of Ayer's examination gives further proof, that Wittgenstein should have attended a CMSC meeting in 1932, although he wasn't officially a member. Ayer's invitation to give a talk to the CMSC was, furthermore, a result of his visit, thus cannot have taken part in 1931. Ayer explicitly mentions the year of his talk at the CMSC, and as we have Wittgenstein's letter to Russell from 1933, it is quite obvious that he attended the meetings despite of his egression.

That Wittgenstein attended Ayer's talk in 1932 is important, because it should be the talk where Wittgenstein and Turing met and seem to have realized a common topic. M.H.A. Newman who was responsible for Alan Turing's Ph.D. thesis and even Alice Ambrose's preparations for the examination in 1935, forced the discussion of Russell's Principia Mathematica (PM) continuously - as a good friend and correspondent of the PM's author. He favoured Ayer as a philosopher with a new view on the PM, driven by a scientific and empirist perspective. Newman was the advisor of Turing's thesis in 1935, and should have indicated Ayer's talk about "Generalisation in Science" to him personally or through Braithwaite. Further evidence for a relationship or affinity of

6 [Dear Gilbert, [...] Wittgenstein is treated here as a second Pythagoras and Weissmann is the high priest of the cult. [...] The news that Wittgenstein has changed his views has however together with other considerations determined me not to bother to revise my article on the Tractatus. Instead I am working to find a definition of 'propositions' from which it will follow formally that all propositions are elementary or truth functions of elementary propositions and a definition of meaning from which it would follow formally that certain metaphysical assertions, e.g. assertions of necessary connexion were meaningless. Then of course we would have to show that the definition were not arbitrary [....]" (Handwritten letter from Ayer to Ryle, dated February 19th, 1933. Gilbert Ryle Notes and Letters. 12 Letters to Ryle. Linacre College, Ryle Cabinet, Oxford.)] Handwritten letter from Ayer to Ryle, dated February 19th, 1933. Gilbert Ryle Notes and Letters. 12 Letters to Ryle. Linacre College, Ryle Cabinet, Oxford.)]

Wittgenstein and Turing is the similarity of opinions, Wittgenstein and Turing explain to have towards Russell:

Wittgenstein introduced his "Philosophy for Mathematicians":

> "Russell's calculus is not fundamental, it is just another calculus. There is nothing wrong with a science before the foundations are laid." (Wittgenstein's Lectures. Cambridge, 1932-1935. Ed. A. Ambrose, Basil Blackwell: Oxford, 1979: 205)

The minutes of Turing's talk at the Moral Science Club on Friday 1st. dec 1933 read:

> "The sixth meeting of the Michaelmas term was held in Mr Turing's rooms in King's College. A.M.Turing read a paper on 'Mathematics and logic'. He suggested that a purely logistic view of mathematics was inadequate; and that mathematical propositions possessed a variety of interpretations, of which the logistic was merely one. A discussion followed. R.B. Braithwaite (signed) (Hodges, Andrew. Alan Turing: the Enigma. London : Burnett Books 1983: 86) [7]

Hodges, Alan Turings biographer, comments on the invitation:

> "Richard Braithwaite, the philosopher of Science, was a young Fellow of King's; and it might well have been through him that the invitation was made. Certainly, by the end of 1933, Alan Turing had his teeth into two parallel problems of great depth. Both in quantum physics and in pure mathematics, the task was to relate the abstract and the physical, the symbolic and the real" (Hodges, Andrew. Alan Turing: the Enigma. London : Burnett Books 1983: 86)

Newman himself gave talks to the society on November 11, 1936, on 'Finitist Mathematics', a review of the situation in metamathematics at that time with reference to the Entscheidungsproblem, on Hilbert, Gödel, and Brouwer (Klagge & Nordmann 2002: 374-375).

Though the mathematicians and philosophers critized Wittgenstein's new way to think about mathematics and logic, on the other hand they accepted his differing approach as something very enlightening. This might be due to the fact, that the anxiety of contradictions in formal languages was low - after Russell had introduced the theory of types - and they otherwise wanted to improve their 'surview' on possible fallacies and paradoxes. Gödel's incompleteness theorem was presented in September 1930 in Königsberg and published 1931[8]. The mathematicians just wondered whether

7 Minutes of University Associations and Clubs, occasionally with related records. Minute Book.. Cambridge University Library: Cambridge University Archives. University/Min. IX. 43. 1926-1935 (1933)

8 In 1930 a conference in Königsstein was held by the Vienna Circle and the "Gesellschaft für Empirische Philosophie" from Berlin (John v. Neumann. Selected Letters. 1903-1957. The American Mathematical Society 2005, p. 8). The conference's subject was the epistemology of natural sciences

they should care about these new problems, and Wittgenstein was one of the philosophers able to make something of it, whereas the mathematicians themselves would just have delivered new theorems:

> "Cantor defined a cardinal number to be the common label of a class of matching sets, and he even showed that one infinite number can be greater than another. The introduction of infinite numbers had some peculiar results, for example, that a number x can be equal to x+1, but for a while it seemed that the new arithmetic, though strange, involved no direct contradiction." (M.H.A. Newman, "Russell's Work in Mathematics".Draft article, typed with handwritten notations. 06th November 1972 The Max Newman Digital Archive, St. John's Library University of Cambridge, Janus Cat. Item 2-15-32: 1)

Newman continues how Burali Forte detected the first paradoxes, that again didn't change the trust in the infinite, until Russell finally showed the antinomies of a class of all classes, that doesn't belong to itself. But again, with Newman, he solved the problem with his theory of types. (Newman 1972: 1)

4 Identity in the Philosophy for Mathematicians

Wittgenstein followed continuously his 'old problem' of the creativity or 'nonsense-likeness' of the identity-sign, but now had **explicitly** new ways to operate on it, that he attached to the current problems within meta-mathematics. He offered not only a surview of different ways to use it, but a new conception that was convincing and in the gist, though the relation to computation failed to be noticed within the process of history.

We see his new concept at work in the Skinner Archive[9] as an identity sign, that just encodes, not

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and mathematics, and on September 7 Gödel announced his later so called First incompleteness theorem in the discussion session of this conference. With Redei (John von Neumann : selected letters. Ed. Rédei, Miklós. American Mathematical Society and London Mathematical Society , xxv, Providence, R.I. & London 2005: 8) John von Neumann immediately grasped the importance of Gödel's theorems, and decided "There is no rigorous justification for classical mathematics" (von Neumann 2005: 8). Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatshefte für Mathematik und Physik 38, 1931, S. 173–198

9 Gibson, Arthur 2010 "The Wittgenstein Archive of Francis Skinner", in: Venturinha, Nuno (ed.), Wittgenstein after his Nachlass, Houndsmills: Macmillan, 2010: 64-77.

Gibson, Arthur Ludwig Wittgenstein: Dictating Philosophy. Cambridge University Press. Forthcoming

Ritter, Ulrike. "Cryptographical Remarks. Wittgenstein's Preference for Seemingly Capricious Rules". Preproceedings 36. Internationales Wittgenstein Symposium, 36. Internationales Wittgenstein Symposium. Kirchberg am Wechsel, 11.-17. August 2013. Revised version forthcoming.

pretends to describe. Even if it is formally recursion on variables embedded in new definitions by equations, it shows, that meaning can be just a code, an assignment, without a necessary descriptive sense - though it might intermingle with a descriptive language-game.

Identity was grammatized explicitly in the Philosophy for Mathematicians in 1932-1935: In den Notes by Ambrose "Philosophy for Mathematicians 1932-35, the expression "grammar" is used for the identity-sign:

> "But what is meant by '1+1=1+1?' It is part of the grammar of '=' that one can write this formula. But how is it used?" (Ambrose 1979: 207)

As already mentioned, in 28th May 1930, he had explained his idea to the Trinity Mathematical Society. (L.C. Young complained "that there was no sense in saying a=a is a convention". Klagge & Nordmann 2002: 373)

Ambrose's transcript shows, that Wittgenstein understood mathematical problems in direct reference to the logical problems he himself had tried to solve since 1914:

> "What Russell has said about number is inadequate, first because criteria for his use of identity are not mentioned in Principia, and secondly because the notation for generality is confusing". (Ambrose 1979: 205)

He specifies, that generalization (quantification) is wrongly understood as referring to (number-) objects, though numbers should be regarded as reducible to occurences of empirical objects in a respective frequency. This is just nominalism, a philosopher might argue, but it is not the whole story of Wittgenstein's way to conceive numbers. More specific, and obviously even technical, is his explanation "*Number is an attribute of a function defining a class; it is not a property of the extension. A function and a list are to be distinguished."*

This implies, that "law and extension [the list, u.R.] are utterly different" (Ambrose 1979: 206) which is especially obvious when infinite lists are concerned. In this context, the term "grammar" is used for mathematical concepts:

> "An 'infinite number' has an entirely different grammar from 'finite number'. We need not define 'infinite number'; rather, we must say how the term is used" (Ambrose 1979: 206).

Wittgenstein shows himself still engaged in the idea of a most rational, justified symbolism and an approach of strict reduction of existence assumptions. He proposes as formal explanation of denotation, avoiding generalization:

"Russell would write 'Only a satisfies f' as (Ex x) (fx and x=a). I would write it as 'fa and Not (Ex x,y).fx.fy'" (Ambrose 1979:206)

In a way, Wittgenstein's suggestion is redundant to Russell's or even contradictory:
Not (Ex x,y) (fx and fy) is equivalent to All x,y Not (fx.fy), which could be instantiated unrestricted because of the universal quantification to (fa -> Not fa) or just Not (fa und fa), which contradicts the simplest laws of logic like idempotency or maintenance of truth. Nevertheless, Wittgenstein seem to have been interested in this very modifications, to prevent the denotata of "a" to be conceived as simple, persistent objects. Of course his ideas transcend even modern mathematical logic, not just the traditional logic or phenomenological concepts. His aim is obviously, in the tradition of the TLP, not just a look at grammar, but an investigation into the possibilities of certain realms of physical-informational phenomena, that do not obviously follow the concepts of identity and object-persistence that he critizes.

However, he tries step by step to avoid a real problem that goes along with the quantification of identity, that is the assumption of the existence of something the variable denotes. This leads to an obvious muddle of contingent and necessary propositions, as a=a is trivially true for all variables, but Ex x : x=a is contingent. He explains in the lectures:

"The x in (Ex x)fx stands for a thing, a substrate; and propositions having different grammars, both mathematical and nonmathematical propositions, are dealt with in the same way." (Ambrose 1979: 205)

Again, in contrast of some very distinguished comparisons in his writings, Wittgenstein looks at the current use of identity in mathematics in a sometimes not convincing way:

"The formula 'a = a' uses the identity sign in a special way; for one would not say that a may be substituted for a. Yet we do start in inductions with something like a=a". (Ambrose 1979: 207).

Doesn't Wittgenstein go wrong with the assumption, that something like a=a is the start of an induction? In a way, we have a further confirmation of how he conceived identity as assignment. "n = 1" means, n is assigned to have the value 1. Similar does "n = n+1" not describe an equality, but assigns 'n+1' to the n of the former calculation.. But mathematics succeeds in this case to distinguish between the variable which might have all values in the class for which n is defined, and the value 1 as a special case. Wittgenstein calls it "substitution", which shows what is right, but does not explain the similarity he sees to 'a=a'. But there is at least one aspect, in which his claim is true concerning identity in mathematical induction: sentences or expressions like a=a are found at the end of an inductive proof. Let's think of a simple example like

We try to show
$\sum n_i = i*n$ (for i = 0 to m, or for simplification, for i= 1 to 5) [for n = 1 and n+1]

We assume n = 1
with the result that
$\sum n_i = 1 + 1 + 1 + 1 + 1 = 5$
and i*n = 5*n = 5.
The result is 5, hence, identical for both expressions in the equation, what proves the assumption (for n).

For n= n+1
$\sum n_i = n+1 +n+1+n+1+n+1+n+1 = 5*n + 5 = 5*(n+1)$
and n*i = (n+1) * 5 for n = n+1

Hence, again the same expressions are visual at the end of an inductive proof.

Wittgenstein rethinks the role of such results-as-proofs and the use of identity several times in his manuscripts of the 30s, and the idea to localise such identity-claims at the beginning of an induction seems not just due to a similarity of assignment and substitution but to the situation of a lecture, too.

Otherwise, the importance of this visualization of identity (seldom it is 'additionally' claimed that e.g. (the result of the left expression) 5*(n+1) = 5*(n+1) (the result of the right expression)

As such, even if his precise reference to the use of identity in induction is not precisely true, it is enlightening, as it shows in the reflection on induction that the identity-sign is already avoided by a kind of order that is given to the resulting expressions on the page where the proof is written. And actually, the notes of Ambrose end with Wittgenstein's statement:

> "What counts in mathematics is what is written down. Symbols obviously do interest even the intuitionist, who says that mathematics is not a science about symbols but about meanings. [...] The intuitionist should be asked to show how 'meaning' operates. (Ambrose 1979: 225)

And at this point Wittgenstein explains that "the final sentence of a demonstration"[...] is part, namely the end, of the proof, and the proof incorporates it into a new calculus". (Ambrose 1979: 222)

5 Incompleteness in the context of hermeneutical and formal interpretation in the early 30s

Such coordinated in his thought, he discusses Russell's antinomy and the Axiom of reducibility (axiom of comprehension), that assumes for every set the existence of another set to be an element of. The operations on symbols, identity as coding-sign (assignment-sign), determines his perspective. Otherwise the difference between "the law" and "the extension" is kept within his philosophy of mathematics as an aspect of the calculus that should not be violated. This tension is 'used' to initialize an appealing comparison of the method of hermeneutics and Gödel's incompleteness theorem in MS 140.[10]

In MS 140, about 1932, that evolved into the Philosophical Grammar and was mentioned as parallel to the 'Philosophy for mathematicians' noted by Ambrose, Wittgenstein contrasts the concept of "verstehen" (to understand) and "Deutung" (interpretation) with the idea that language is "incomplete" (incomplete) and conceals truth. I like to emphasise another aspect. Wittgenstein finishes his remark:

> "Eine Interpretation ist doch wohl etwas, was in Zeichen gegeben wird. Es ist diese Interpretation im Gegensatz zu einer anderen (die anders lautet). Wenn man also sagen wollte - "jeder Satz bedarf noch einer Interpretation", so hieße das: kein Satz kann ohne einen Zusatz verstanden werden."
> "To be sure, an 'interpretation' is something that is given in signs. It is this interpretation, as opposed to another (which reads differently). So if one wanted to say 'Every proposition needs an interpretation' that would mean: No proposition can be understood without a rider". (Ludwig Wittgenstein, Philosophical grammar : part I, The proposition, and its sense, part II, On logic and mathematics. Ed. Rhus Rhees. Berkeley : University of California Press 1974: § 9. ; Wittgensteins Nachlass. The Bergen Electronic Edition. Bergen & Oxford UP: 2000MS 140 jpg.10; Ms 114, jpg.18; Big Typescript § 4, jpg.15)

Whereas Derrida would jump into this supplement dechainé, I prefer to take the subjunctive into account and try to understand what Wittgenstein means we should prefer to this possibilty of understanding interpretation as a chain of supplemented signs, and where exactly his implied criticism points to.

We have already discussed the idea of a visual identity of an interpretation, that continues from the ideal symbolism of the TLP into the philosophy and the grammar and use of signs he investigates. In formal inductive reasoning, as in mathematical equations generally, the seemingly senseless identity-sign is used or 'substituted' by similar results with similar surfaces (like 5(n+1)), which are the result of introducing a term into a calculus. As "the law differs completely from the extension"

10 The awareness of the Big Typescript and the typical essemblage of contextually different concepts is due to a talk of David Stern, "The Middle Wittgenstein Revisited", 36. Internationales Wittgenstein Symposium. Kirchberg am Wechsel, 11.-17. August 2013.

(Ambrose 1979: 205) , it seems not to matter whether the extension is the same or not, we might take it for granted in respect to the calculus.

In the central remark in MS 140, §9 (Wittgensteins Nachlass, 2000, MS 140, jpg 9 und jpg 10.) Wittgenstein offers his idea of computing numbers and variables in a visual calculus, a chart, in the context of questions that use hermeneutical concepts like "understanding" and "Deutung". He provokes confusion, how these traditional concepts of theology and poetics relate to the formal and technical perspective that is attached to mathematical logic. Hence, "Deutung" is not just "interpretation", but the analysis, construction, interpretation, the understanding, conceived at least on the surface as denoting something that is hidden as reference of the text:

> "Ich könnte auch sagen: Es scheint uns, als ob wir dem Befehl durch das Verstehen etwas hinzufügen, was die Lücke zwischen Befehl & Ausführung füllt. Das heißt doch, was den Befehl in schattenhafter Weise ausführt. So daß wir dem, der sagt, "Aber du verstehst ihn ja, er ist also nicht unvollständig" antworten können,"Ja, aber ich verstehe ihn nur, weil ich noch etwas hinzufüge: die Deutung nämlich".

> "I could also say: It seems to us that by understanding the command we add something to it that fills the gap between the command and its execution. And surely that means, something that executes the command in a shadowy way. So that to someone who says 'But you do understand it, so it isn't incomplete,' we can answer: 'Yes, but I understand it only because I add something to it: namely, the interpretation".(MS 140, Wittgensteins Nachlass 2000, jpg17 & jpg18)

The translation fails in a way to show the difference between the german word "Deutung" and the german-english word "interpretation". "Deutung" is definitely not used for any mathematical context in German, and even "understanding" and "command" are evidently concepts that are in tension to each other. As Wittgenstein has in 1932 already declared to look at the use of concepts, it seems to be ignorant not to think about this strange combination of words as something in need of an explanation. This is of special importance, as not just terms concerning understanding are used, but the questions on the whole focuses on the possible incompleteness of understanding or interpretation. Incompleteness was a vibrant concept in the philosophy of logic and mathematics of the early 30s. In 1931, Kurt Gödel had published his article on the incompleteness of arithmetics[11], which immediately insecured the mathematicians in respect of the reliability of their foundations, after the antinomies Russell detected working on his PM (Principia Mathematic) had been more or less solved with the theory of types and the axiom of selection.

Hence, we should ask how "Deutung" and "interpretation", "incompleteness" and "understanding",

11 Gödel, 1931: 173–198

as "to understand" and "executing a command" enlighten each other - not to forget that the chart itself shows a *computable* system of defined fields for results - and as we have a computable function for all IN in x^2 (x ∈ IN), it is not obvious whether the finity of the numbers given in the chart is not just for technical reasons of the drawing.

Gödel however, starts with a syntactically finite sequence of natural numbers and the presupposition, that the deduction rules of PM and ZF are sufficient to decide all mathematical question expressible in those systems. His proof shows, "*that this is not true, but that there are even relatively easy problems in the theory of ordinary whole numbers that can not be decided from the axioms*". (Kurt Gödel: Über formal unentscheidbare Sätze der Principia Mathematica und verwandter Systeme I. Monatshefte für Mathematik und Physik 38, 1931: 2)

Gödel introduces a class K, such that K := {n ∈ IN / not provable (R_n(n))}
R_n(n) is a class sign for any relation (a provable formula) that correlates an n ∈ IN to a number attached by diagonalisation, with the number having the same symbol as itself.
The relation R is definable and satisfiable and recursive. But nevertheless we cannot decide whether R_n(n) is provable for a respective R_n or not - though this seems plausible to assume if we can rely on the properties of definability, satisfiability and recursion, that should supply to calculate the properties of all possible formulas in the system.

One reason Gödel gives is the liar, that emerges by a reflective use. As "not provable R_n" and "provable R_n" are well-formed formulas, we can enumerate them by diagonalisation with "not provable R_n"=1 and "provable R_n"=2. Hence, the Gödel-number of our K is 1 and $K_n \in R_n$(n), and provable R_1(1). Hence, it is provable that for every given n (1 or 2) R_n is provable, which leads to the contradiction that K (1) is assigned to itself ("not provable R_1") iff it is provable that it is assigned to itself and thus not "not provable", hence wrongly assigned to itself: 1 ∈ K_1 iff 1 not ∈ K_1. In this version of the paradox, the provability is shown by existing diagonalisations that create a truth, that R assignes 1 to K_1, and a proof, that it can not be the case, because it implies to be unprovable though it is obviously provable by the diagonalisation itself.

Again, we have on the first sight an argument from 'vision' - an existing diagonalisation on differently looking formulas, and the visual correlations as proof of some of the implied assumptions. Hence, Gödel's argument follows the line of Wittgenstein's idea cited ahead: *"To be sure, an 'interpretation' is something that is given in signs. It is this interpretation, as opposed to another (which reads differently)*"

But before we assign a fitting Gödel-number in an existing diagonalisation, to the set K, it might just be empty. Then, we have "not provable" just as an intension or modality without existence, but not as a property in the sense of something that is capable to build a set (not a class), using comprehension. Hence, regarding Wittgenstein's declaration, that the law differs from the extension, we are not forced to assume a truth that is unprovable, as the diagonalisation might just be conceived as wrongly relating to a pseudo-class with K_1=1, if the formula(s) for the empty set have already been diagonalized with 3, e.g, and thus just K_1 =3 (it is confessed, that K_1=0 or something likewise looks more comfortably common) would be a right application of the diagonalization-rule.

In this respect, Gödel's incompleteness could be revisited as implying creativity. This seems what Wittgenstein has divined. He criticized the model of interpretation that is used in Russell's PM as conceiving mathematical entities as objects, denoted by signs. It goes along with denotation within diagonalisation, that the enumerated formulas look like different sets, like objects.

Exactly the effect Wittgenstein again and again argues against, the inference to existence from the identity of a sign, is visible in Gödel's argument:

$K_1 = 1$

$\rightarrow$ Ex K ($K_g = 1$)

Otherwise, if we do not distinguish between just possible classes and sets, what seems not be conceived as necessary by Gödel, we can have truths without provability, like ◊ Ex K ($K_g = 1$) or just ◊ ($K_g = 1$) what just means that we can imagine a respective method of diagonalization. I think our concept of these truths is more similar to the truths of hermeneutic and understanding, that emerge within a realm of concepts we can syntactically build and understand in respect to the methods we have for understanding - obviously this might reduce to a 'simpler language' that just uses some axioms and definitions to explain recursion and hence the general conception we have of a concept. The sentences of a formal language of this kind would not even be decidable by their unique coherence to the base sentences of such a system, and imply not any necessary connection to the world at all (comp. the discussion of modality, and espec. necessity and disquotationalism at the beginning of this paper).

Hence, in this way, understanding and executing a command converge, and as such, to distinguish between formulas with bounded variables that can be true or false and formulas that merely 'exist' in a system to sustain certain transformations. It is quite obvious that Wittgenstein disliked this muddling of object-like denotata, seemingly denoting expressions and sheer intension. He changed it at least towards an investigation how the things, which we frequently classify as signs, are used to encode other signs or objects to operate.

Otherwise, he confesses in a way - and hence does not give up -, that quantification and infinity have to be reducible to a continuing practise or get other evidence, like the visual evidence for equality we see in operations that result in the same signs, and which no machine would additionally ask for being really equal or not. So if we are not, but do not have any privileged relation to a corresponding reality either, how can we explain when we know that an operation is to be continued and another is not ?

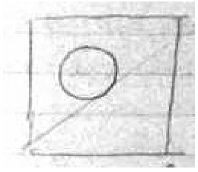
As already shown, Wittgenstein rejects the seemingly trivial use of identity and the rule of quantification already in the notebooks and diaries in 1914. He was led to this radical decision in respect to mathematics, that depends in a way on the equality of terms, by the idea of an ideal symbolism, wherein every variable denotes something real, and no quantification is necessary. The TLP tried, in a way, to present an image of the world that is caused by ideal, undistorted perception and thought.

The rejection of identity and quantification is also related to the ambigious value of modal truth the identity statement for a variable and its quantification have. For a = a, the existence of a denotatum of "a" is required, of a model, as otherwise, "a" is merely a sign without persistence, though the existence is (then) declared for the sign itself. But 'Ex x (x = a)' conflicts with the fact that a is just a variable and even if within an ideal symbolism variables should denote, it is not even sure or un-ambigious, whether the existence refers to a special denotatum or to the sign itself, as if it was an usual object. While it seems harmless and even important that the signs of a formal syntax are identical and persistent like objects, this presumption lead to the creation of objects and a muddling of reference-relations. These do not necessarily lead to confusion (as such a universe is merely possible), and might even by enlightening (like the great narrations in the culture of our world), but just in case the symbolism is regarded as such as related to possibilities. Wittgenstein at least prefers to improve the symbolism and his concept of language even in the 30s, and to avoid identity and symbol-persistence for a special kind of coding rules using identity. Though the idea of a real field of possibilities limited just by methods like diagonalisation, sorting, calculating, shifting a.s.o. shines through, in the thirties, Wittgenstein gives more emphasis to a more restricted world of contingent truths, dependent on given methods of calculation and use. But within the just negative attempt to give up identity he frees his considerations of necessity and evolves into a concept of language and its objects and facts determined by assignment rules and related to methods and operations we have to operate on signs. The difference becomes visible if we think of disquotation, as explained in relation to the problems regarded in the TLP. The alusion to hermeneutics in MS 140 leaves open if the methods and operations we could use to substitute the necessity of a disquotation-equivalence, are restricted to the ones we already have and know. And obviously not even accepted operations lead to a persistent universe, as the shift of identity of expressions or their denotata is (at least possibly) implied by every step.

He conceives the identity of a character, which does not require the persistence of variables but regulates it. It is comparable to an assignment operator in computer science. Existence is thus reduced to a current possibility, but no longer follows as a necessary property, as the necessary property to be persistent is in Wittgenstein's philosophy not any longer assumed. This step was essential as improving the applicability of mathematics and as a step towards computation.

6 Visualization and criteria for following a rule

The manuscripts of the 30s show how Wittgenstein searched for visual models for calculations and logical considerations, that are comparable to the geometrical evidence that is given in e.g. drawings of intersecting lines, which show their common point to be their common point. He tries to find examples that transfer this kind of evidence to logic.

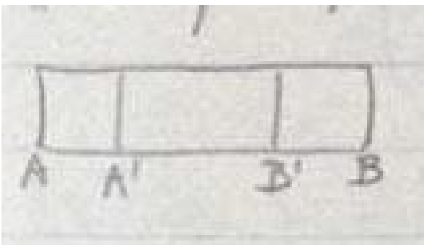


"Wenn man die allgemeinen Sätze von der Art 'Der Kreis befindet sich im Quadrat' betrachtet, so kommt es einem immer wieder so vor als sei die Angabe der Lage im Quadrat nicht eine nähere Bestimmung zur Angabe der Kreis liege im Quadrat (Wenigstens nicht, so weit der Gesichtsraum in Betracht kommt) als sei vielmehr das 'im Quadrat' eine komplette Bestimmung die an sich nicht mehr näher zu *bestimmen* sei." (MS 109, 007.jpg)

(If we look at the general sentences of such kind like 'The circle is placed within the square', so we again and again get the impression that the information about how the circle is situated in the square was not a more precise determination about the circle's place in the square (At least not as far as the visual space is concerned) as if just the other way round the 'in the square' was a complete determination which could not be more precise." (Transl. UR).

This example shows how he tries to get a better insight into generality in mathematical expressions. If the circle is just 'in the square', without further specification, it can be anywhere in the square. That implies an infinity of possible positions, and nevertheless we can completely understand what is meant. The circle in the square has, additionally, one similarity to the subsets of set-theory. It is not just visual, what 'in the square' means and how it might vary, but the implication of something that is not itself a geometrical object could be explained in this way, too.

Just some pages after this drawing on page 3 of MS 109, on page 7 (MS 109, 011.jpg), a drawing of stripes follows. Wittgenstein discusses the question, whether we can use expressions like 'inference' to describe what we see in the picture:



"Ist es nicht vielmehr so, daß aus 'der Streifen von A bis B ist weiß' folgt, 'der Streifen A'B' ist weiß', wenn in dem Streifen AB eben die Striche A' und B' gezogen waren. Unendlich ist nur die Möglichkeit dieser Art Figuren (Linienzüge)."

("Is is not rather the case, that from 'the stripe from A up to B is white' follows, 'The stripe A'B' is white', if within the stripe AB these strokes A' and B' were just drawn. Infinite is merely the possibility of this kind of figures (lines).") (Wittgensteins Nachlass, 2000, MS 109)

In Wittgenstein's comment, the search for logical structures within the visual is again obvious, and the topic of infinity is explicit. Furthermore, his examples are quite convincing. The visual structures are such, that they would allow simple operations like moving from field to field, with sections that are logically connected. Turing's sketch of the later so called Turing-machine in his paper from 1936/1937 ("On computable numbers") is quite similar:

> "A machine can be constructed to compute the sequence 010101.... The machine is to have the four m configurations 'b', 'c', 'k', 'e' and is capable of printing '0' and '1'. The behaviour of the machine is described in the following table in which 'R' means 'the machine moves so that it scans the square immediately on the right of the one it was scanning previously'. Similarly for 'L'. 'E' means 'the scanned symbol is erased' and 'P' stands for 'prints'. This table (and all succeeding tables of the same kind) is to be understood to mean that for a configuration described in the first two columns the operations in the third column are carried out successively, and the machine then goes over into the m-configuration described in the last column." (Turing, Alan. "On computable numbers, with an application to the Entscheidungsproblem'" P*roceedings of the London Mathematical Society*, (Ser. 2, Vol. 42, 1937: 233)

Again, we find a similarity in the application to decidability, though Turing presents his idea just as an automatic diagonalization, not as a proof. Whereas Wittgenstein might have created an operation that proves how the operation of diagonalisation has been done, Turing's solution is just an operation on the result of the given diagonalisation: After all solvable formulae are proved, "*Sooner or later K will reach either U or -U. If it reaches U, than we know that U is provable. If it reaches -U, then, since K is consistent (Hilbert and Ackermann, p.65), we know that U is not provable.*" (Turing, 1937, 234). The philosophical impacts of Gödel's theorem retain untouched, as the way Turing applies his computation to the problem does not reach the diagonalisation itself (which would complement Wittgenstein's perspective), but operates just on its results.

How Wittgenstein conceives the role and capacities of such automatical operations, 'mechanism' and mechanical machines, is difficult to evalue, as even the idea of a wire-based field maintenance of (logical) truth is not really explicit in his writings (but so clearly just a possible 'instantiation' of the TLP abstract drawings), though he investigated the logical structure of pictures and drawings in a way that leads to the most famous examples. Even if 'grammatical', the visual evidence of the 'synthetical a priori' of geometry or other visual forms with logical structures - Wittgenstein investigates an unsorted awful lot of them - bring with them a kind of decisiveness in their regularity, as the continuity of following a rule in a visual sense - like drawing strokes on a paper sheet - is decisively denied at the margins of the paper sheet. What comes along with the idea to stop the application of a rule because such a limitation was ignored (like a contradiction in mathematics), gets a kind of clarification if we look at proofs and how they are presented in symbols, how they are written or build as a system of signs, in comparison with applications of such rules, e.g. the square-equations applied on a biological system or the visual evidence a respective parabola gives, or sequences of wire circuits, which are continuously activated, as described at the beginning of this article.

Hence, how can the comparison of formal languages and hermeneutics lead to new insights? The hermeneutical interpretation in the classical sense of a text, e.g. of a mythical or religious text, uses names that do not denote or merely denote just possibilities. The literary-mythic text intends to create in the reader's mind a denotation of its names.

With the discourse or narration, emerge possible denotata, for which the discourse, the texts and their traditions, do not offer independent criteria of decision, whether they are true in a corresponding sense or not. Instead, the difference becomes less obvious for formal languages if they don't restrict themselves explicitly to accepted assumptions of a given basis. Thus, quantification is perhaps likewise difficult for formal languages as for languages that are dependent on aspects of understanding, and hence, Wittgenstein developes the idea of 'going along with rules' - that might very well change everything all over repeatingly. The new use of the identity sign as an assignment operator of a progressive encoding, especially obvious in the Skinner Archive and visual already in the TLP, supplies criteria: It proves to be misunderstood or wrongly applied if it can not be continued (e.g, if stepwise sequences that should remain in a fixed field, exceed this nevertheless. This would show, that the used rule of progressing was wrong.) The criteria for keeping related to something like 'the world' is then, not to end up in false pictures or just not to have to stop in an ongoing picture without wanting to, but be able to continue such a 'conditional branching'.

Thus, what seems to lead into an artificial world of supplements, is obviously quite different entangled in the evidence given by applications and visual models, which go beyond the Kantian synthetical a priori of geometry into a visual and operational evidence of applied logic.

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